POSSIBILITIES OF USING THE NONLINEAR FEM MODEL FOR PARTIALLY LOADED AREAS FOR HIGH STRENGTH CONCRETE

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Abstract

This paper is about possibilities of using the nonlinear finite element method concrete model for partially loaded areas. The model is based on several theories connected with compressive behaviour, smeared cracking and the influence of confinement. The theoretical model is compared with experimental results from tests conducted by Bonetti [1]. The compressive strength of concrete is around 53 MPa and 75 MPa. These results are also compared with EC2 [2] provisions. It is shown that results from calculations according to EC2 are less accurate than from proposed model. However, the FEM model gives results which are too high. This model was previously tested on normal concrete and it gives the average error around 8%. Here it is 26% and 31% depending on the strength of concrete.

Keywords

High strength concrete, nonlinear FEM, partially loaded area

1 INTRODUCTION

Concrete structures are the most popular across the world. During last decades the possibilities of producing the high strength concrete (HSC) have increased. It is beneficial to use the HSC, because it allows to reduce dimension of cross-sections of the designed element and hence it reduces a weight of a structure. The compressive strength of HSC should exceed about 50 or 60 MPa. The mechanical properties are not the same as for normal strength concrete. One of the main differences is a ductility of that concrete. For HSC, the higher compressive strength is, the more brittle concrete is.

Reinforced concrete structures can work in several load states. The best recognised ones are bending, shear and compression. In some structures there is a problem of partially loaded areas. It occurs especially in the anchorage zones of prestressed concrete structures or in same column-column joints. In this areas very high compressive pressure, which is often higher than compressive strength, works on a small limited area.

Most of building codes have some provision about partially loaded areas. Some of them advise to check this area for compressive stresses and also by Strut-and-Tie model (STM). For example European Code for concrete structures (EC2) [2] gives a formula (1) to calculate the maximum compressive forced which can be applied on limited area.

$$F_{Rdu} = A_{c0} \cdot f_{cd} \cdot \sqrt{\frac{A_{c1}}{A_{c0}}} \tag{1}$$

where:

F_{Rdu} – maximal compressive force,

f_{cd} – design compressive strength,

A_{c0} – loaded area,

A_{c1} – distribution area.

EC2 also advises to reduce this force in case high shear or unequally spread pressure. The code does not give the method how to take into consideration this influence.

Since the second half of 20th century in structural engineering the finite element method (FEM) is used for designing. The technological development of computer and increase in their speed allowed to build more complex models. For simple engineering situations linear analysis is sufficient. For more complicated ones it is good to use nonlinear methods.

In the regions where very high compressive forces act the concrete works in the triaxial stress state. To analyse this problem nonlinear FEM can be used. Several FEM models can be applied. The one which is presented in this paper bases on several theories, connected with compressive behaviour, smeared cracking and influence of different aspects. This model was used for normal strength concrete and give quite good approximation of the ultimate strength of concrete. The average error was about 6%. The work on this subject is still not published.

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2 DISCRIPTION OF THE SPECIMENS

Bonetti in his work [1] prepared 7 series of specimens. Two of them where made of HSC. In experiment he used concrete cylinders (Fig. 1) with diameter of 152.4 mm (6 in.) and height of 304.8 mm (8 in.). The only one difference between series was the compressive strength of concrete. In the first one it was 53.16 MPa and in the second one it was 75.49 MPa. The cylinder where put into press machine. The bottom surface of the specimen was supported on the whole area. The load was applied from the top by the press through the steel square or circular or hexagonal plate, depending on the planned ratio (R) of top surface area to loaded area. This paper is concentrated on ratios R from 2.0 to 6.0, where the square steel plates were used. The numerical specimens where similar to the experimental ones. Due to the symmetry of the loads and the shape, it was decided to use half of cylinder. It reduced the time of numerical analysis.

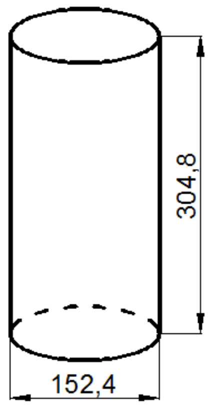

Fig. 1 *Experimental specimen*

3 DISCRIPTION OF NONLINEAR FEM MODEL

The numerical analysis was done in TNO DIANA program with FX+ (graphic interface), which can be used for this kind of work. The model was prepared with the usage of functions implemented in the program. Compressive behaviour was defined by the Madrid parabola. The strain-stress formulas are (2) and (3). After reaching strain which corresponds with compressive strength, the strain-stress relation was constant until reaching ultimate concrete strain, which was calculated by formula from EC2 [2].

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \qquad \text{for } 0 \le \varepsilon_c \le \varepsilon_{c2} \tag{2}$$

$$\sigma_c = f_{cd} \qquad \text{for } \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \tag{3}$$

where:

σ_c – compressive stress in concrete,

ε_c – actual strain in concrete,

ε_{c2} – the smallest strain for which concrete achieves its compressive strength,

ε_{cu2} – ultimate strain of concrete in uniaxial compression,

n – is 2 for normal concrete, and for concrete of $f_{ck} \ge 50$ MPa $n = 1.4 + 23.4[0.01(90-f_{ck})]^4$.

The values of ε_{c2}, ε_{cu2}, n for concrete of compressive strength 53.16 MPa are 2‰, 3.5‰ and 2 respectively and for concrete of compressive strength 75.49 are 2.39‰, 2.69‰ and 1.46. The parabolas (Fig. 2) were constructed as a multiline with small steps.

In the model also an impact of confinement was taken into account. It was done by using model proposed by Selby and Vecchio [3].

The value of Young modulus was calculated using formulas from Model Code 2010 [4]. Tensile strength of concrete was not presented in Bonetti's work [1]. It was calculated also from formulas of Model Code 2010 [4]. The tension softening behaviour was set according to proposition of Hordijk, Cornelissen and Reinhardt [5, 6]. For this aim the fracture energy was needed. It was also calculated according to Model Code 2010 [4].

The smeared cracking model is a total strain model which consist of assumptions which are mentioned above. To make more accurate calculations an impact of lateral cracking a relationship prepared by Vecchio and Collins [7] was used.

A presented model include formulations which respects of reduction of Poisson's ratio, where the concrete is cracked.

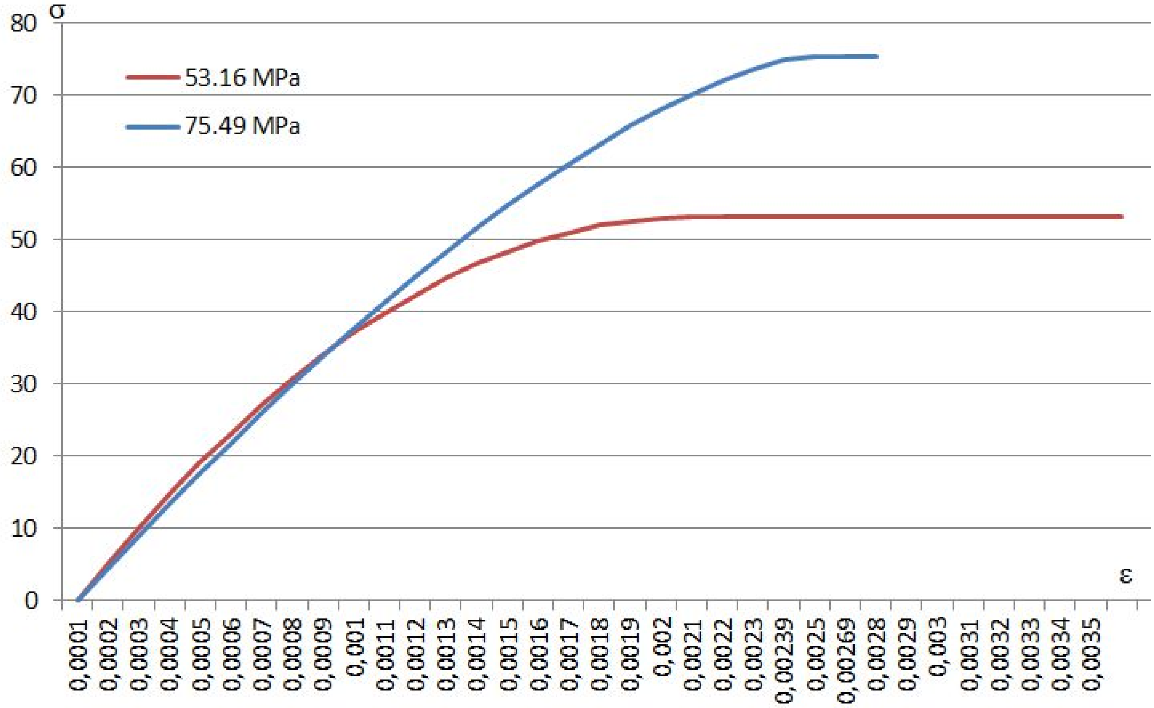

Fig. 2 *Relation of σ-ε for concrete*

The elements was set as pyramids with linear interpolation. The mesh of specimens has size of 12mm (for R = 6.0 – size 10mm). It gives about 2000 and 3200 nodes.

The numerical specimens were halves of cylinder. They were supported on the bottom surface in vertical direction. The surface which was create by cutting the cylinder was supported in normal direction. Three nodes on the bottom surface on the symmetry line were pinned in the perpendicular direction to this symmetry line. The meshed specimen is shown in Fig. 3.

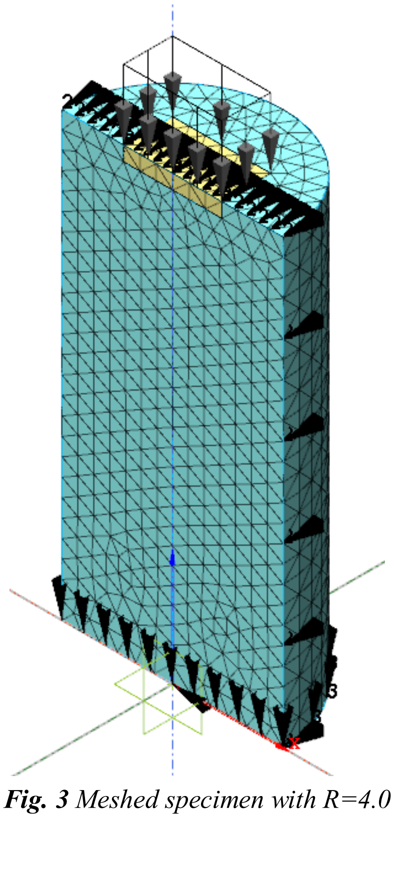

Fig. 3 *Meshed specimen with R=4.0*

Load was applied to each specimen as uniform pressure on the square area on the top surface. The load was increased in maximum steps of 1.0 MPa to minimum 0.1 MPa. The exact size of the step was calculated as iteration method and was dependent on the optimal number of iterations in one step (it was 7 iterations) and the number of iterations in the previous step. The maximal number of iterations was 50. If convergence was not achieved, it meant that specimen exceeded its capacity. The convergence was checked for: displacement with 0.01 limit, force with 0.01 limit and energy with 0.001 limit.

4 PROVISIONS BY EUROCODE 2 [2]

Provisions which are given by EC2 give a possibility to calculate bearing capacity of partially loaded areas due to concrete crushing. The maximal force which can be applied can be obtained by using formula (1). In this formula there is a value of A_{c1} which is called a distribution area. EC2 defines this area as a area which is similar to the load area and its center in the line of acting force. It can be seen on Fig. 4. Also dimensions of that area cannot be bigger more than 3 times than dimensions of the loaded area.

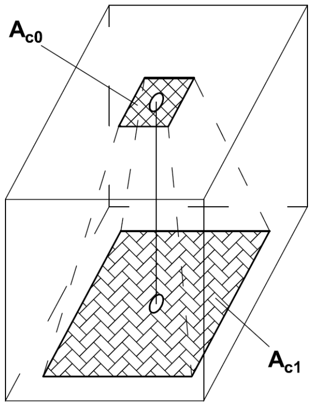

***Fig. 4** Loaded area (A_{c0}) and distribution area (A_{c1})*

The condition of the similarity of the loaded and distribution causes that the distribution area is smaller than the circular cross section of the specimen and has a square shape. Because of that the value R is not the same as A_{c1}/A_{c0}.

In this paper comparison of result from EC2 [2], numerical analysis and experiments was done.

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f_c [MPa]	*53.16*				
R [-]	2.0	2.5	3.0	4.0	6.0
$A_{c1}/A_{c0} < 3.0$	1.27	1.59	1.90	2.54	3.00
q_{exp} [MPa]	53.42	56.55	61.83	62.44	85.60
q_{FEM} [MPa]	66.08	71.10	77.26	80.88	107.50
q_{EC2}[MPa]	67.68	84.76	101.53	135.24	159.48
n_{FEM}	1.24	1.26	1.25	1.30	1.26
n_{EC2}	1.26	1.50	1.64	2.16	1.86

***Tab. 1** Results for concrete of 53.16 MPa compressive strength*

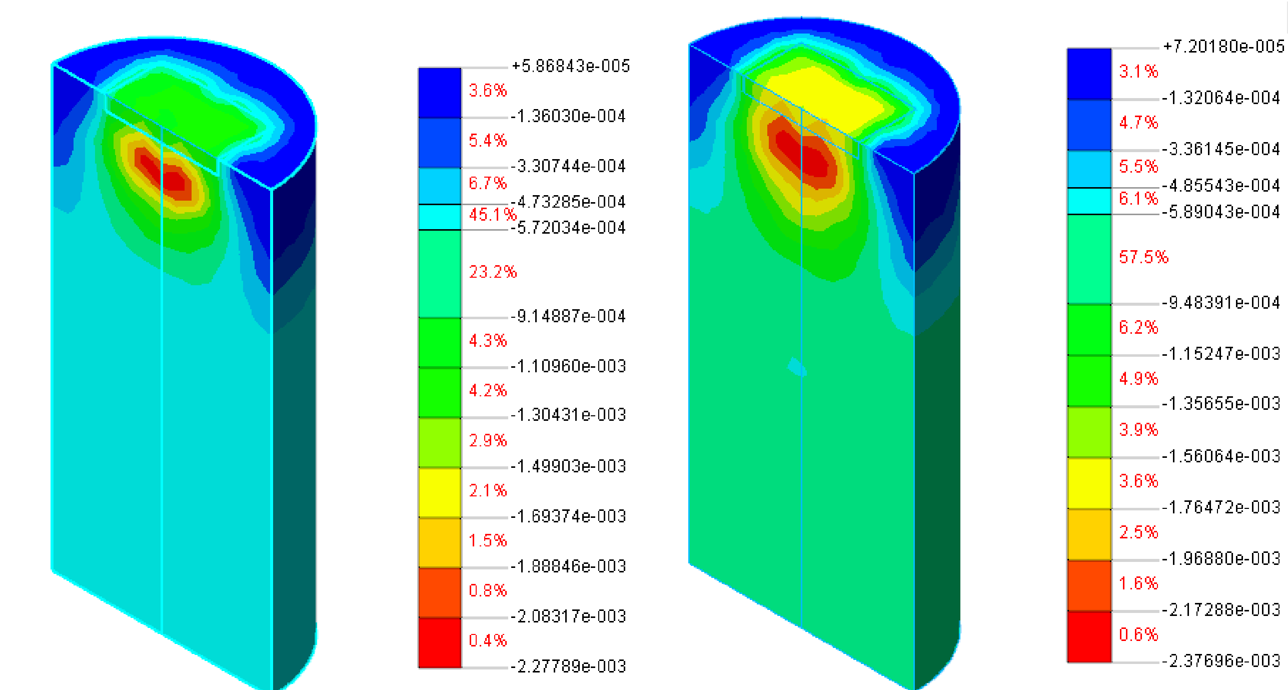
5 RESULTS

The results of test for concrete of 53.16 MPa and 75.49 MPa compressive strength are shown in Tab. 1 and Tab. 2 respectively. The presented value q are the values of pressure on the loading surface at the moment of failure. The indexes: “exp” means experimental, “FEM” means from FEM analysis and “EC2” means according to EC2 provisions on crushing of a partially loaded concrete. The value n_{FEM} is the ratio q_{FEM}/q_{exp} and n_{EC2} is q_{EC2}/q_{exp}.

The compressive strain just before failure for 53.16 MPa concrete was between 0.0020 and 0.0025 and was rising with the increase a value of R. For 75.49 MPa concrete it was from 0.0022 to 0.0031. These value are shown as examples on specimens with R = 3.0 in Fig. 5.

f_c [MPa]	*75.49*				
R [-]	2.0	2.5	3.0	4.0	6.0
$A_{c1}/A_{c0} < 3.0$	1.27	1.59	1.90	2.54	3.00
q_{exp} [MPa]	61.21	67.10	76.08	85.45	119.28
q_{FEM} [MPa]	86.09	91.64	97.80	111.40	139.00
q_{EC2}[MPa]	96.11	120.35	144.17	192.03	226.46
n_{FEM}	1.41	1.37	1.29	1.30	1.16
n_{EC2}	1.57	1.79	1.89	2.24	1.90

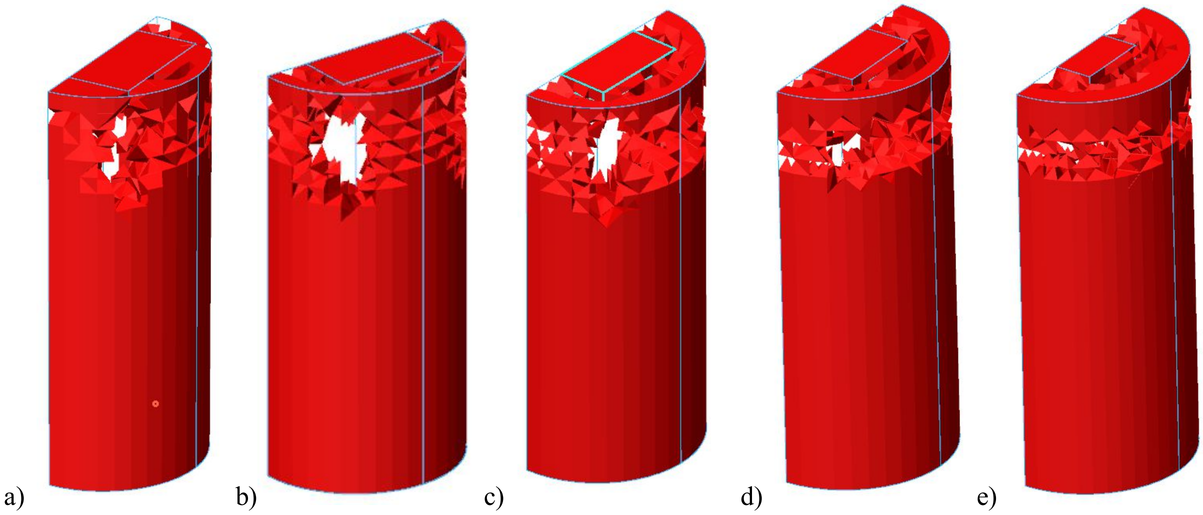
***Tab. 2** Results for concrete of 75.49 MPa compressive strength*



a) b)

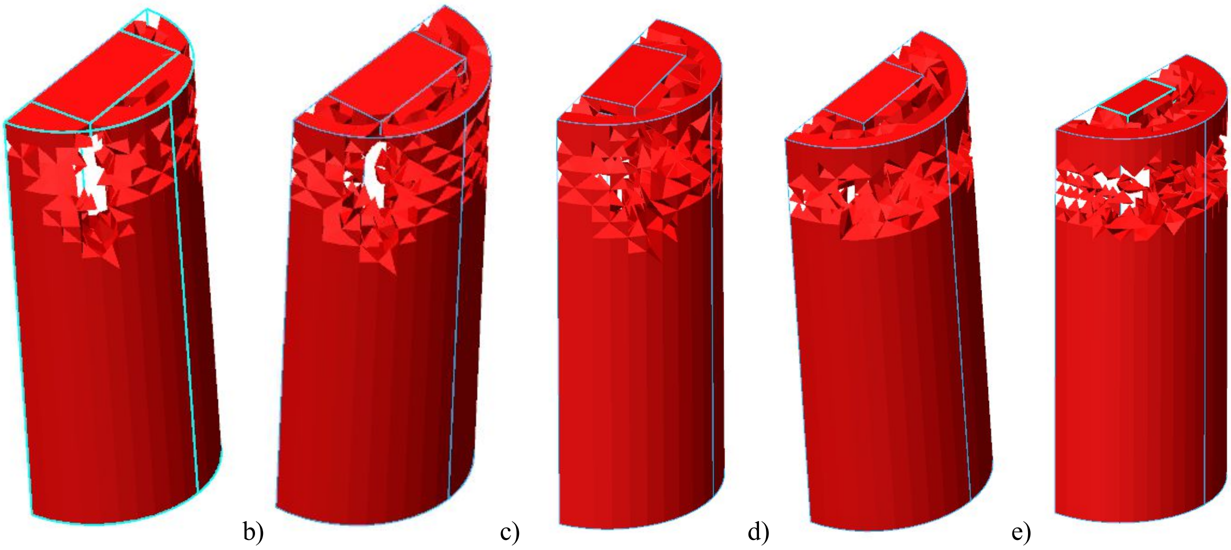
***Fig. 5** Strains in vertical direction just before failure (R=3.0: a) 53.16 MPa concrete, b) 75.49 MPa concrete*

The regions which are cracked or crushed just nearly before the failure in each specimen are presented (as empty volume) in Fig. 6 for concrete 51.16 MPa and on Fig. 7.



a) b) c) d) e)

***Fig. 6** Cracked and crushed regions in 53.16 MPa concrete specimens:*
a) R=2.0, b) R=2.5, c) R=3.0, d) R=4.0, e) R=6.0



a) b) c) d) e)

***Fig.** 7 Cracked and crushed regions in 75.49 MPa concrete specimens:
a) R=2.0, b) R=2.5, c) R=3.0, d) R=4.0, e) R=6.0*

6 ANALYSIS OF RESULTS

Calculated by using FEM results are higher than obtained in experimental results. The average error for the 53.16 MPa concrete is around 26% and for the 74.49 MPa concrete is about nearly 31%. Results according EC2 [2] are also higher than those from experiments, but the average errors are much evan than numerical ones. They are 69% for the 53.16 MPa concrete and around 88% for the 75.49 MPa concrete. Comparing the results with those from paper on normal strength concrete which was mentioned in introduction, it must be said that this model and EC2 provisions work worse for high strength concrete. Nevertheless the number of tested specimens is too low. For low R the value of q is lower than f_c. This fact is hard to understand.

The strains just before the failure are higher than those which are in concrete at the moment of achieving stresses equal to concrete strength in uniaxial compression (ε_{c2}). This strains were not obtained experimentally.

From Figs. 6 and 7 it can be easily noticed that specimens with lower ratios R (2.0, 2.5 and 3.0) have significant vertical cracks on the line which goes through the centre of cylinder to the corner of the loaded area. Nonetheless these cracks are also present on specimens with higher R. These specimens are more cracked on tinier region around the specimen. These cracking centreline is about 6 cm below the top of the specimens. There is also bigger cracking just on the top surface, next to the loaded area for higher ratios R. These observations are similar for both types of concrete. It is difficult to decide from which concrete specimens are more cracked.

7 CONCLUSIONS AND PROPOSITION FOR FURTHER RESEARCHES

Presented data show that the proposed model and EC2 provisions are not very consistent with experimental results. However the amount of tested specimens is not sufficient. Due to this observation it is important to be very careful while using EC2 provisions for HSC. Nonetheless the model shows cracking of the specimens quite well. The possibility of reducing concrete strength by some factor is reasonable because the single errors of the specimens were similar to average one.

The interesting fact is that specimens with low values of R (e.g. 2.0 or 2.5 for 75.49 MPa concrete) fail at the lower level of pressure than in uniaxial strength test.

According to general knowledge on the partially loaded areas the values from test seems to be quite accurate. The effect of concrete confinement was possible to notice.

In further researches it is planned to modify the compressive behaviour. Also others theories for tension softening will be tested. The author consider to conduct experimental test on bigger number of specimens from HSC. These test should be done especially on the values of R which are between 1 and 3, because they often occur in anchorage zones.

What is more, the impact of opening for HSC concrete should be experimentally and numerically tested. Also the influence of a spiral as reinforcement is going to be tested.

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